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Cosmological Models with Variable "Constants"

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The behavior of the "constants," G, c, $\hbar$, a, e, m_i, and Λ, considering them as variable, in the framework of a flat cosmological model with FRW symmetries described by a bulk viscous fluid and considering mechanisms of adiabatic matter creation are investigated. Two cases are studied; one with radiation predominance and another of matter predominance. It is found that with the solution obtained our model verifies these basic principles: Lorentz invariance and general covariance, Mach, Equivalence and causality. Finally, to emphasize that the envisaged models are free from the main problem: Planck's, horizon and entropy. With regard to that model with matter predominance it is seen that mechanisms of creation of matter cannot be considered since if these are taken into account the temperature would increase instead of remaining constant while the universe expands.

1. INTRODUCTION

In a recent paper [1] the behavior of the "constants" G, c, and Λ was investigated within a model described by a bulk viscous fluid, and taking into account mechanisms of matter creation, to solve the entropy problem. Upon considering the constant c as a function dependent on time t, the condition that the radiation constant a should be constant in the same way that Boltzmann's constant k_B was imposed. With this supposition the following is obtained: Planck's constant $\hbar$ should behave as $\hbar \propto c^{-1}$. In this paper this point is taken up once more but without similar hypothesis, i.e., Boltzmann's constant k_B is the only constant considered real. Therefore, we suppose that all the "constants" G, c, $\hbar$, a, e, m_i, and Λ are variable, without making any previous hypothesis about their behavior or verifying any equality in particular. Calculations are made within the framework described above, regarding their behavior together with the rest of the quantities which characterize the model: f; ρ; ρ_m; θ; S; s; ξ; and n where, respectively, they represent the radius

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of the universe; energy density; matter density; temperature; entropy; entropy density; viscosity coefficient; and particle number density.

Once all these quantities are calculated, two concrete solutions are studied: one that it would describe a universe with radiation predominance and another with matter predominance, simplifying both solutions to the case of noncreation of matter, i.e., $\beta = 0$. It is found, for example, that with these solutions, it is always verified that G/c^2 (general covariance) stays constant in both cases, independently of the value of β. The expression $\rho = a\theta^4$ is recovered for energy density. All energies are preserved, but not the moment in the case of matter predominance, while in the case of radiation predominance, the energy follows the law $E \propto t^{-1/2}$ while the moment is constant. The fine structure constant α in both cases, continues being a real constant in spite of the fact that all the constants that define it vary. The models described here verify the following basic principles: as already indicated general covariance; it is also shown that the principle of Lorentz invariance is verified, Mach, Equivalence; and causality. Both models lack the designated horizon problem since the relationship: $f = ct$ is always verified. With the solutions obtained, it is seen that the model lacks the designated problem of Planck in the same way as that of entropy.

The paper is organized as follows: In the second section the governing equations of our model are shown and considerations on the dimensional method followed, are made. In the third section, use is made of the D.A. (Pi theorem) to obtain a solution to the principal quantities that appears in the model. Finally, in the fourth section presentation is made of two particular cases of the obtained solutions together with some conclusions.

2. THE MODEL

For a flat universe $k = 0$ with FRW symmetries, i.e., homogeneity and isotropy were assumed and therefore there will be no solely temporary spatial variations of "constants" G, c, and Λ. It is also supposed that our fluid is bulk viscous (second viscosity) and mechanisms of creation of matter are considered. With these suppositions the equations that govern the model are as follows:

$$2\frac{f''}{f} + \frac{(f')^2}{f^2} = -\frac{8\pi G(t)}{c^2(t)}(p + p_c) + c^2(t)\Lambda(t) \tag{1}$$

$$3\frac{(f')^2}{f^2} = \frac{8\pi G(t)}{c^2(t)}\rho + c^2(t)\Lambda(t) \tag{2}$$

$$n' + 3nH - \psi = 0 \tag{3}$$

where n measures the particles number density, ψ is the function that measures the matter creation, $H = f'/f$ represents the Hubble parameter (f is the scale factor that appears in the metrics), p is the thermostatic pressure, ρ is energy density, and p_c is the pressure that generates the matter creation.

The creation pressure p_c depends on the function ψ. For adiabatic matter creation this pressure takes the following form [2]:

$$p_c = -\left[\frac{\rho + p}{3nH}\psi\right] \tag{4}$$

The state equation used is the known expression

$$p = \omega\rho \tag{5}$$

where $\omega = const.$ $\varphi \in [0, 1]$ physically realistic equations, thus the energy-momentum tensor T_{ij} verifies the energy conditions.

It is necessary to know the exact form of the function ψ, which is determined from a more fundamental theory that involves quantum processes. It is assumed that this function follows the law:

$$\psi = 3\beta nH \tag{6}$$

here we are following to Lima *et al.* [2] (for other treatment [3] while Prigogine *et al.* [4] follows this other law $\psi = \kappa H^2$) where β is a dimensionless constant (if $\beta = 0$ then there is no matter creation since $\psi = 0$), presumably given by models of particles physics of matter creation.

The conservation principle brings us to the following expression:

$$\rho' + 3(\omega + 1)\rho\frac{f'}{f} = (\omega + 1)\rho\frac{\psi}{n} \tag{7}$$

Integrating the equation (7) the following relationship between energy density and the radius of the universe is obtained and even more important the constant of integration necessary for our subsequent calculations:

$$\rho = A_{\omega,\beta} f^{-3(\omega+1)(1-\beta)} \tag{8}$$

where $A_{\omega,\beta}$ is the constant of integration that depends on the state equation that is considered, i.e., of the constant ω and of the constant β that measures the matter creation.

The effect of the bulk viscosity in the equations is shown replacing p by $p - 3\xi H$ where ξ follow the law $\xi = \xi_0\rho^\gamma$ (see [5], [6], and [3]). This last state equation, in our opinion, does not verify the homogeneity principle for this reason it is modified by

$$\xi = k_\gamma\rho^\gamma \tag{9}$$

where the constant k_γ causes this equation to be dimensionally homogeneous for any value of γ.

The dimensional analysis followed needs to make the following distinctions: it is necessary to know beforehand the set of fundamental quantities together with that of unavoidable constants (designated as governing parameters in the nomenclature of Barenblatt). In this case, the only fundamental quantity that appears in the model is the cosmic time t as can be easily deduced from the homogeneity and isotropy supposed for the model. The unavoidable constants of the model are the constant of integration $A_{\omega,\beta}$ that depends on the state equation ω and of the mechanisms of matter creation β and the constant k_γ that controls the influence of the viscosity in the model.

In a previous work [7] the dimensional base was calculated for this type of model, being this $B = \{L, M, T, \theta\}$ where θ represents the dimension of the temperature. The dimensional equation of each one of the governing parameters are:

$$[t] = \mathrm{T} \qquad [A_{\omega,\beta}] = L^{3(\omega+1)(1-\beta)-1}MT^{-2}$$
$$[k_\gamma] = L^{\gamma-1}M^{1-\gamma}T^{2\gamma-1}$$

All the derived quantities or governed parameters in the nomenclature of Barenblatt will be calculated in the function of these quantities (the governing parameters), that is to say, in function of the cosmic time t and of the two unavoidable constants k_γ and $A_{\omega,\beta}$ with respect to the dimensional base $B = \{L, M, T, \theta\}$.

3. SOLUTIONS THROUGH D.A.

Calculation will be made through dimensional analysis D.A., i.e., applying the Pi Theorem, the variation of $G(t)$ in function of t, the speed of light $c(t)$, the Planck's constant $\hbar(t)$, the radiation constant a, the charge of the electron $e(t)$, the mass of an elementary particle m_i, the variation of the cosmological "constant" $\Lambda(t)$, the energy density $\rho(t)$, the matter density $\rho_m(t)$, the radius of the universe $f(t)$, the temperature $\theta(t)$, the entropy $S(t)$ and the entropy density $s(t)$, the viscosity coefficient $\xi(t)$, and, finally, the particle number density $n(t) \propto f^{-3}$.

The dimensional method brings us to (see [7] and [8]).

3.1. Calculation of $G(t)$

As indicated above, will be accomplished calculation of the variation of G applying the Pi theorem. The quantities considered are: $G = G(t, k_\gamma, A_{\omega,\beta})$, with respect to the dimensional base $B = \{L, M, T, \theta\}$. We know that $[G] = L^3M^{-1}T^{-2}$

	G	t	k_γ		A_ω
L	3	0	$\gamma - 1$	$3(\omega + 1)(1 - \beta)$	-1
M	-1	0	$1 - \gamma$		1
T	-2	1	$2\gamma - 1$		-2

we obtain a single monomial that leads to the following expression for G

$$G \propto A_{\omega,\beta}^{\frac{2}{3(\omega+1)(1-\beta)}} k_\gamma^{\frac{2+3(\omega+1)(1-\beta)}{3(\omega+1)(1-\beta)(\gamma-1)}} t^{-4-\left[\frac{2+3(\omega+1)(1-\beta)}{3(\omega+1)(1-\beta)(\gamma-1)}\right]} \tag{10}$$

3.2. Calculation of $c(t)$

$c(t) = c(t, k_\gamma, A_{\omega,\beta})$ where $[c] = LT^{-1} \Rightarrow$

$$c(t) \propto A_{\omega,\beta}^{\frac{1}{3(\omega+1)(1-\beta)}} k_\gamma^{\frac{1}{3(\omega+1)(1-\beta)(\gamma-1)}} t^{-1-\left[\frac{1}{3(\omega+1)(1-\beta)(\gamma-1)}\right]} \tag{11}$$

3.3. Calculation of Planck's Constant $\hbar(t)$

$\hbar = \hbar(t, k_\gamma, A_{\omega,\beta})$ where the dimensional equation is: $[\hbar] = L^2MT^{-1} \Rightarrow$

$$\hbar(t) \propto A_{\omega,\beta}^{\frac{1}{(\omega+1)(1-\beta)}} k_\gamma^{\frac{1-(\omega+1)(1-\beta)}{(\omega+1)(1-\beta)(\gamma-1)}} t^{\frac{(\omega+1)(1-\beta)[1+(n-1)]-1}{(\omega+1)(1-\beta)(\gamma-1)}} \tag{12}$$

3.4. Radiation Constant $a(t)$

$a = a(t, k_\gamma, A_{\omega,\beta}, k_B)$ where $[a] = L^{-1}MT^{-2}\theta^{-4} \Rightarrow$

$$k_B^{-4}\, a(t) \propto A_{\omega,\beta}^{\frac{-4}{(\omega+1)(1-\beta)}} k_\gamma^{\frac{-4+3(\omega+1)(1-\beta)}{(\omega+1)(1-\beta)(\gamma-1)}} t^{\frac{4-3(\omega+1)(1-\beta)}{(\omega+1)(1-\beta)(\gamma-1)}} \tag{13}$$

3.5. Charge of the Electron $e(t)$

$e = e(t, k_\gamma, A_{\omega,\beta}, \epsilon_0)$ where $[\epsilon^2\epsilon_0^{-1}] = L^3MT^{-2} \Rightarrow$

$$e^2(t)\epsilon_0^{-1} \propto A_{\omega,\beta}^{\frac{4}{3(\omega+1)(1-\beta)}} k_\gamma^{\frac{4-3(\omega+1)(1-\beta)}{3(\omega+1)(1-\beta)(\gamma-1)}} t^{\frac{-4+3(\omega+1)(1-\beta)}{3(\omega+1)(1-\beta)(\gamma-1)}} \tag{14}$$

3.6. Mass of an Elementary Particle $m_i(t)$

$m = m(t, k_\gamma, A_{\omega,\beta})$ where $[m] = M \Rightarrow$

$$m(t) \propto A_{\omega,\beta}^{\frac{1}{3(\omega+1)(1-\beta)}} k_\gamma^{\frac{1-3(\omega+1)(1-\beta)}{3(\omega+1)(1-\beta)(\gamma-1)}} t^{2-\left[\frac{1-3(\omega+1)(1-\beta)}{3(\omega+1)(1-\beta)(\gamma-1)}\right]} \tag{15}$$

3.7. Cosmological Constant $\Lambda(t)$

$\Lambda = \Lambda(t, k_\gamma, A_{\omega,\beta})$ where $[\Lambda] = L^{-2} \Rightarrow$

$$\Lambda(t) \propto A_{\omega,\beta}^{\frac{-2}{3(\omega+1)(1-\beta)}} k_\gamma^{\frac{-2}{3(\omega+1)(1-\beta)(\gamma-1)}} t^{\frac{2}{3(\omega+1)(1-\beta)(\gamma-1)}} \tag{16}$$

3.8. Calculation of Energy Density $\rho(t)$

$\rho = \rho(t, k_\gamma, A_{\omega,\beta})$ with respect to the dimensional base B, where $[\rho] = L^{-1}MT^{-2}$

$$\rho \propto k_{\gamma}^{\frac{1}{1-\gamma}} t^{\frac{1}{\gamma-1}} \tag{17}$$

it is observed that this relationship shows that energy density does not depend either on the state equation ω or on the mechanisms on creation of matter, i.e., it does not depend on the constant $A_{\omega,\beta}$ solely on the viscosity of the fluid.

3.9. Matter Density $\rho_m(t)$

$\rho_m = \rho_m(t, k_\gamma, A_{\omega,\beta})$ where $[\rho_m] = ML^{-3} \Rightarrow$

$$\rho_m(t) \propto A_{\omega,\beta}^{\frac{-2}{3(\omega+1)(1-\beta)}} k_{\gamma}^{\frac{-2-3(\omega+1)(1-\beta)}{3(\omega+1)(1-\beta)(\gamma-1)}} t^{2-\left[\frac{-2-3(\omega+1)(1-\beta)}{3(\omega+1)(1-\beta)(\gamma-1)}\right]} \tag{18}$$

3.10. Calculation of the Radius of the Universe $f(t)$

$f = f(t, k_\gamma, A_{\omega,\beta})$ where $[f] = L \Rightarrow$

$$f \propto A_{\omega,\beta}^{\frac{1}{3(\omega+1)(1-\beta)}} k_{\gamma}^{\frac{1}{3(\omega+1)(1-\beta)(\gamma-1)}} t^{\frac{-1}{3(\omega+1)(1-\beta)(\gamma-1)}} \tag{19}$$

It can be observed that

$$q = -\frac{f''f}{(f')^2} = -1 - 3(\omega + 1)(1 - \beta)(\gamma - 1)$$

$$H = \frac{f'}{f} = -\left(\frac{1}{3(\omega + 1)(1 - \beta)(\gamma - 1)}\right)\frac{1}{t}$$

3.11. Calculation of the Temperature $\theta(t)$

$\theta = \theta(t, k_\gamma, A_{\omega,\beta}, k_B)$ where k_B is the Bolztmann constant: $[\theta] = \theta$ and $[k_B\theta] = L^2MT^{-2} \Rightarrow$

$$k_B\theta \propto A_{\omega,\beta}^{\frac{1}{(\omega+1)(1-\beta)}} k_{\gamma}^{\frac{1-(\omega+1)(1-\beta)}{(\omega+1)(1-\beta)(\gamma-1)}} t^{-\left[\frac{1-(\omega+1)(1-\beta)}{(\omega+1)(1-\beta)(\gamma-1)}\right]} \tag{20}$$

3.12. Calculation of the Entropy $S(t)$

$S = s(t, k_\gamma, A_{\omega,\beta}, a)$ where a is the radiation constant. $[S] = L^2MT^{-2}\theta^{-1}$

$$S \propto A_{\omega,\beta}^{\frac{1}{(\omega+1)(1-\beta)}} k_{\gamma}^{\frac{1-\frac{3}{4}(\omega+1)(1-\beta)}{(\omega+1)(1-\beta)(\gamma-1)}} t^{-\left[\frac{1-\frac{3}{4}(\omega+1)(1-\beta)}{(\omega+1)(1-\beta)(\gamma-1)}\right]} a^{\frac{1}{4}} \tag{21}$$

3.13. Calculation of the Entropy Density $s(t)$

$s = s(t, k_\gamma, A_{\omega,\beta}, a)$ where a is the radiation constant. $[s] = L^{-1}MT^{-2}\theta^{-1}$

$$s \propto A^{0}_{\omega,\beta} k_{\gamma}^{\frac{3}{4(\gamma-1)}} t^{-\left[\frac{3}{4(\gamma-1)}\right]} a^{\frac{1}{4}} \tag{22}$$

3.14. Calculation of the Viscosity Coefficient $\xi(t)$

$\xi = \xi(t, k_\gamma, A_{\omega,\beta})$ where $[\xi] = L^{-1}MT^{-1}$

$$\xi \propto k_{\gamma}^{\frac{1}{1-\gamma}} t^{\frac{-\gamma}{\gamma-1}} \tag{23}$$

3.15. Particle Number Density $n(t)$

$n = n(t, k_\gamma, A_{\omega,\beta})$ where $[n] = L^{-3}$ obtaining

$$n(t) \propto A_{\omega,\beta}^{\frac{-1}{(\omega+1)(1-\beta)}} k_{\gamma}^{\frac{-1}{(\omega+1)(1-\beta)(\gamma-1)}} t^{\frac{1}{(\omega+1)(1-\beta)(\gamma-1)}} \tag{24}$$

4. DIFFERENT CASES

All the following cases can be calculated without difficulty. But as indicated in the first section, attention is centred only on those models that follow the law $\xi = k_\gamma \rho^{1/2}$, i.e., $\gamma = (1/2)$, which corresponds to models that are topologically equivalent to the classic FRW [9]. Two models with $\gamma = (1/2)$ are studied: one with $\omega = 1/3$ which corresponds to a universe with radiation predominance and another with $\omega = 0$ corresponding to a universe with matter predominance.

4.1. Model with Radiation Predominance $\gamma = 1/2$ and $\omega = 1/3$

$$\begin{aligned}
G &\propto A_{\omega}^{\frac{1}{2(1-\beta)}} k_{\gamma}^{-2-\frac{1}{(1-\beta)}} t^{-2+\frac{1}{(1-\beta)}} \\
c &\propto A_{\omega}^{\frac{1}{4(1-\beta)}} k_{\gamma}^{\frac{-1}{2(1-\beta)}} t^{-1+\frac{1}{2(1-\beta)}} \\
\hbar &\propto A_{\omega}^{\frac{3}{4(1-\beta)}} k_{\gamma}^{2-\frac{3}{2(1-\beta)}} t^{\frac{1+2\beta}{2(1-\beta)}} \\
k_B^{-1/4} a &\propto A_{\omega}^{\frac{-3}{(1-\beta)}} k_{\gamma}^{\frac{6\beta}{1-\beta}} t^{\frac{6\beta}{\beta-1}} \\
e^2 \epsilon_0^{-1} &\propto A_{\omega}^{\frac{1}{(1-\beta)}} k_{\gamma}^{\frac{-2\beta}{1-\beta}} t^{\frac{2\beta}{(1-\beta)}} \\
m &\propto A_{\omega}^{\frac{-1}{2(1-\beta)}} k_{\gamma}^{2-\frac{1}{2(1-\beta)}} t^{\frac{1}{2(1-\beta)}} \\
\Lambda &\propto A_{\omega}^{-\frac{1}{2(1-\beta)}} k_{\gamma}^{\frac{1}{(1-\beta)}} t^{-\frac{1}{(1-\beta)}}
\end{aligned}$$

With these results it is proven that the relationship G/c^2 (general covariance) remains constant without the need of imposing it as other authors do [10] and [11].

$$\frac{G}{c^2} = \frac{t^{-2+\frac{1}{(1-\beta)}}}{t^{-2+\frac{1}{(1-\beta)}}} = const. \tag{25}$$

Likewise, it is observed that the fine structure constant remains constant independently of the value of β

$$\alpha = \frac{e^2}{\epsilon_0 c\hbar} = \frac{t^{\frac{2\beta}{(1-\beta)}}}{t^{-1+\frac{1}{2(1-\beta)}} t^{\frac{1+2\beta}{2(1-\beta)}}} = const.$$

that is to say, in this model a possible variation of the fine structure constant α cannot be explained [12]. In the way in which the variation of the charge of the electron has been calculated it cannot be discerned whether ϵ_0 is constant or not. Let us suppose that $\epsilon_0 = const.$ (for an opposite point of view [13] and the appendix) therefore $e \propto t^{\frac{\beta}{(1-\beta)}}$ and from the relationship $c^2 = \dfrac{1}{\epsilon_0\mu_0}$ is obtained $\mu_0 \propto c^{-2} \propto t^{2-\frac{1}{(1-\beta)}}$.

If $\beta = 0$ becomes (there is no matter creation) the following results are found:

$$G \propto t^{-1}, \qquad c \propto t^{-1/2}, \qquad \hbar \propto t^{1/2}, \qquad a = const.$$
$$e^2\epsilon_0^{-1} = const., \qquad \mu_0 \propto t, \qquad m_i \propto t^{1/2}, \qquad \Lambda \propto t^{-1}$$

The result of $G \propto t^{-1}$ is very well-known in the literature. The value of $c \propto t^{-1/2}$ also has been obtained by Troiskii [14] and Barrow [15] (in very different contexts). A similar result to $\hbar \propto t^{1/2}$ can be found in [16], [17], [18], and [19] whereas for a contrary point of view see [20]. The constancy or not of the charge of the electron and of the fine structure constant have been discussed (amongst others) by [21]. $e^2\epsilon_0^{-1} = const.$ has been obtained in particular if it is assumed that $\epsilon_0 = const.$ then it is determined that $e = const.$ and $\mu_0 \propto t$, with respect to $G \propto t^{-1}$ and $m_i \propto t^{1/2}$. A similar result to that is obtained by Hoyle and Narlikar [22] and Canuto *et al.* [23]. A study on the implications of the variation of the masses can be found in Mansfield *et al.* [24].

With respect to rest of the quantities, the same behavior is obtained as that of Lima *et al.* [2], except for the temperature θ and the particle number density n.

$$\rho \propto k_\gamma^2 t^{-2}, \qquad \rho \propto t^{-2}$$
$$k_B\theta \propto A_\omega^{\frac{3}{4(1-\beta)}} k_\gamma^{2-\frac{3}{2(1-\beta)}} t^{-2+\frac{3}{2(1-\beta)}}$$

$$
\begin{aligned}
f &\propto A_{\omega}^{\frac{1}{4(1-\beta)}} k_{\gamma}^{-\frac{1}{2(1-\beta)}} t^{\frac{1}{2(1-\beta)}} \\
a^{\frac{-1}{4}} S &\propto A_{\omega}^{\frac{3}{4(1-\beta)}} k_{\gamma}^{-\frac{3\beta}{2(1-\beta)}} t^{\frac{3\beta}{2(1-\beta)}} \\
a^{\frac{-1}{4}} s &\propto A_{\omega}^{0} k_{\gamma}^{\frac{3}{2}} t^{-\frac{3}{2}} \\
\xi &\propto k_n^2 t^{-1} \\
n &\propto A_{\omega}^{\frac{-3}{4(1-\beta)}} k_{\gamma}^{\frac{3}{2(1-\beta)}} t^{\frac{-3}{2(1-\beta)}}
\end{aligned}
$$

In the results, the temperature θ depends explicitly on β on such a way that $\beta < \frac{1}{4}$ so that the temperature of our universe does not increase. On the contrary it cools down as it expands. With this value of β, $q > \frac{1}{2}$ is obtained. The same happens with the value obtained for n. The result depends explicitly on the parameter β. The rest of the quantities coincide with those obtained by Lima *et al.* except, obviously, for the quantity ξ since their model describes a perfect fluid. With these solutions, our model does not have the horizon problem posed by classic FRW since $ct = f$.

With respect to the thermodynamic behavior, the matter creation formulation considered here is a clear consequence of the nonequilibrium thermodynamic in the presence of a gravitational field. We see that the β parameter works in the opposite sense of the expansion, that is, reducing the cooling rate with respect to the case where there is no matter creation. A very meaningful result is the fact that the spectrum of this radiation cannot be distinguished from the usual blackbody spectrum at the present epoch (see [2]). Therefore, models with adiabatic matter creation can be compatible with the isotropy currently observed in the spectral distribution of the background radiation. At the same time, it can be observed that the obtained model is clearly irreversible (the classic FRW is reversible).

With these results the following law for the energy density

$$\rho = a\theta^4$$

is recovered since the radiation constant a also depends on β and obviously the law $\rho f^{4(1-\beta)} = const.$ is verified. It should be pointed out, furthermore, that the variation of the entropy is due to the variation of the radiation constant a. The energy follows the law

$$E = \hbar\nu \propto t^{\frac{-1+4\beta}{2(1-\beta)}}, \qquad E = m_i c^2 \propto t^{\frac{-1+4\beta}{2(1-\beta)}}$$

since $E = k_B\theta$.

It should also be mentioned that all the important quantities of the classic FRW models are recovered if $\beta = 0$ is made [7]:

$$f \propto t^{1/2}, \qquad \rho \propto t^{-2}, \qquad \theta \propto t^{-1/2}, \qquad S = const.$$
$$s \propto t^{-3/2}, \qquad n \propto t^{-3/2}$$

It is interesting to mention that the model presented here may significantly alter the predictions that the classic FRW makes on the abundance of elements. Such a result possibly limits the values that could be taken by the β parameter.

Finally and according to Prof. Alfonso-Faus's works ([25]), the desire is to show that the model exposed verifies, in addition to general covariance principle (25), the principles of Lorentz invariance, Mach, and Equivalence.

The Lorentz invariance is verified if the relationship v/c remains constant with time

$$\frac{v}{c} = const.$$

but this relationship is always constant since in our model all the speeds vary following the law $v \propto t^{-1/2}$.

Mach's principle is verified if the following equality is fulfilled

$$\frac{GMm}{f(t)} = mc^2$$

this establishes the equality between the energy of a particle and the gravitational potential energy of the same. It can be proved without difficulty that even β is verified.

Finally, the Equivalence principle is verified if the following relationship

$$\frac{GM}{f^2(t)}\frac{t}{c} = const.$$

is maintained constant. It can be proven without difficulty that it is verified even for all values of β.

With regard to Planck's system ([26]), we now shows the following behavior:

$$l_p = \left(\frac{G\hbar}{c^3}\right)^{1/2} \approx f(t)$$
$$m_p = \left(\frac{c\hbar}{G}\right)^{1/2} \approx f(t)$$
$$t_p = \left(\frac{G\hbar}{c^5}\right)^{1/2} \approx t$$

with this behavior it is seen that this model does not have the designated problem of Planck since the radius of the Universe $f(t)$ in the Planck's era coincides with the length of Planck

$$f(t_p) \approx l_p$$

while energy density in the Planck's era coincides with energy density of Planck

$$\rho(t_p) \approx \rho_p \approx t^{-2}$$

where $\rho_p = m_p c^2/l_p^3$.

4.2. Model with Matter Predominance $\gamma = 1/2$ and $\omega = 0$

$$\begin{aligned}
G &\propto A_\omega^{\frac{2}{3(1-\beta)}} k_\gamma^{-2-\frac{4}{3(1-\beta)}} t^{-2+\frac{4}{3(1-\beta)}} \\
c &\propto A_\omega^{\frac{1}{3(1-\beta)}} k_\gamma^{\frac{-2}{3(1-\beta)}} t^{-1+\frac{2}{3(1-\beta)}} \\
\hbar &\propto A_\omega^{\frac{1}{(1-\beta)}} k_\gamma^{\frac{-2\beta}{(1-\beta)}} t^{\frac{1+\beta}{1-\beta}} \\
k_B^{-1/4} a &\propto A_\omega^{\frac{-4}{(1-\beta)}} k_\gamma^{\frac{2+6\beta}{1-\beta}} t^{\frac{2+6\beta}{\beta-1}} \\
e^2 \epsilon_0^{-1} &\propto A_\nu^{\frac{4}{3(1-\beta)}} k_\gamma^{1-\frac{8}{3(1-\beta)}} t^{\frac{2+6\beta}{3(1-\beta)}} \\
m_i &\propto A_\omega^{\frac{1}{3(1-\beta)}} k_\gamma^{2-\frac{2}{3(1-\beta)}} t^{\frac{2}{3(1-\beta)}} \\
\Lambda &\propto A_\omega^{-\frac{2}{3(1-\beta)}} k_\gamma^{\frac{4}{3(1-\beta)}} t^{-\frac{4}{3(1-\beta)}}
\end{aligned}$$

With these results it is seen that exactly the same occurs as in the case of radiation predominance, i.e., that is, the relationship G/c^2 (general covariance) remains constant and the fine structure constant also remains constant in this case. It is easily proven that in the same way as in the case of radiation predominance this model also fulfills the principles studied previously: Equivalence; Mach; and Lorentz invariance.

If $\beta = 0$ is made:

$$G \propto t^{-\frac{2}{3}}, \qquad c \propto t^{-1/3}, \qquad \hbar \propto t, \qquad a \propto t^{-2}$$
$$e \propto t^{\frac{1}{3}}, \qquad m_i \propto t^{\frac{2}{3}}, \qquad \Lambda \propto t^{-4/3}$$

is obtained: $c \propto t^{-1/3}$ is also obtained by Barrow [15] but not by Troiskii [14]. We observe that with $\beta = 0$ this time the charge of the electron behaves as $e^2\epsilon_0^{-1} \propto t^{\frac{2}{3}}$ if $\epsilon_0 = const.$ is considered, then $e \propto t^{\frac{1}{3}}$ while $\mu_0 \propto t^{2/3}$. The radiation constant also varies $a \propto t^{-2}$. The masses continue varying in proportion to time while the gravitation "constant" varying as $G \propto t^{-\frac{2}{3}}$. Finally it should be pointed out that the Planck's constant varies direct proportion to time $\hbar \propto t$.

The rest of the quantities presents the following behavior:

$$\rho \propto k_\gamma^2 t^{-2}$$
$$\rho_m \propto A_\omega^{\frac{-2}{3(1-\beta)}} k_\gamma^{2+\frac{4}{3(1-\beta)}} t^{\frac{-4}{3(1-\beta)}}$$
$$f \propto A_\omega^{\frac{1}{(1-\beta)}} k_\gamma^{-\frac{2}{3(1-\beta)}} t^{\frac{2}{3(1-\beta)}}$$
$$\xi \propto k_\gamma^2 t^{-1}$$

The law of temperatures obtained is:

$$k_B\theta \propto A_\omega^{\frac{1}{(1-\beta)}} k_\gamma^{\frac{-2\beta}{(1-\beta)}} t^{\frac{2\beta}{(1-\beta)}}$$

as the temperature in the matter predominance era should be kept constant, the only possibility is to make $\beta = 0$, i.e., during this era there is no creation of matter (in the case of not making $\beta = 0$ our Universe would be heated). With $\beta = 0$, it is proven that energies are preserved

$$E = \hbar\omega = const. \qquad E = mc^2 = const.$$

since $E = k_B\theta = const.$ This model with $\beta = 0$ is very similar to a FRW with matter predominance though here the problem of the horizon no exits since $ct = f$,

$$\rho \propto t^{-2} \qquad \rho_m \propto t^{-4/3} \qquad f \propto t^{2/3}$$

With respect to the obtained result with the parameter $q = 1/2$, it might seem be in contradiction with the current observed values (acceleration of the Universe) which are based on the constancy of the luminosity of stars. However, in our case the luminosity varies in inverse proportion to time

$$L \propto \frac{GMm_pc}{\sigma_T} \approx t^{-1}$$

where m_p represents the proton mass and σ_T is the cross-section, i.e., the luminosity decreases with time. Sandage has calculated the effect of the evolution on the luminosity of galaxies.

$$\frac{L'}{L} = 10^{-9}/year$$

In our case

$$\frac{L'}{L} = t^{-1}$$

and for an age of the Universe about 10^{10} *years* does not disagree of our result.

Before ending a reference should be made to Petit's work [10].

This author, in a very different context, gauge invariance, has studied the variation of the physical constant, being one of the first to consider the possible variation of the constant c [27]. His results coincide with ours for the case: ($\gamma = 1/2$, $\omega = 0$, $\beta = 0$), i.e., an universe topologically equivalent to the classic FRW with matter predominance and without creation of matter. However, Petit says to work with a universe that describes the radiation era, said coincidence does not exist with our work. However, we believe that his work is very correct in the development, but in reality, he is describing a universe with matter predominance by using in all his work the mass density (see equation number (32) in [10]). For this reason his model cannot verify the law $\rho \propto f^{-4}$. If it is assumed (in our opinion) that his model describes a universe with matter predominance, it is found that all his results coincide with ours for the case above described, i.e., ($\gamma = 1/2$, $\omega = 0$, $\beta = 0$), these are:

$$G \propto t^{-\frac{2}{3}}, \qquad c \propto t^{-1/3}, \qquad \hbar \propto t, \qquad a \propto t^{-2},$$

$$e \propto t^{\frac{1}{3}}, \qquad m_i \propto t^{\frac{2}{3}}, \qquad \epsilon_0 = const., \qquad \mu_0 \propto t^{2/3}$$

$$f \propto t^{2/3} \qquad \rho_m \propto t^{-4/3}$$

Recently, P. Midy and Petit [28] have elaborate a "very interesting" 5D new model where all the "constants" vary in the same way as here.

5. CONCLUSIONS

The behavior of the "constants" within two specific models has been calculated. In the first of the cases, a universe with radiation predominance, it has been seen that the mechanisms of matter creation are valid provided that $\beta < 1/4$, since of the contrary our universe would be heated as it expands. If we restrict ourselves to the case $\beta = 0$ (noncreation of matter) the solutions obtained are not discordant with those already obtained by other authors. In this case, it is found that the radiation constant as well as the relationship $e^2\epsilon_0^{-1}$ remain constant if $\beta = 0$ while the rest of the "constants" vary independent of the value of β. The two models studied here verify the general covariance principles G/c^2, Lorentz invariance $v/c = const.$, Mach, and Equivalence for all value of β. It is also found that the fine structure constant α remains constant since the "constants" that define it vary in such a way that the relationship remains constant. To emphasize, furthermore, that with the variation of the constant of radiation a the relationship $\rho = a\theta^4$ is recovered for energy density. Finally, it should be commented that this model upon varying the speed of the light does not have the problem of the horizon, being verified the equality $ct = f$. It has also been possible to explain the so-called Planck's problem as well as the entropy problem.

With respect to the model with matter predominance, it is seen that in it mechanisms of creation of matter cannot be considered, since if these are taken in account the temperature would increase instead of remaining constant while the universe expanded. With $\beta = 0$, it is proven that energies are preserved. In this case, the same as in the previous, we see that the relationship G/c^2 remains constant the same as the fine structure constant α. But if $\gamma \neq 1/2$, these relationships do not stay constant. Finally it should emphasized that in this case, contrary to what happened in the model of radiation predominance, with $\beta = 0$ the "constants" a and e vary.

6. APPENDIX. BEHAVIOR OF THE ELECTROMAGNETIC QUANTITIES

Following Prof. Alfonso-Faus's observations we explore other possibilities from the results we have obtained for the product $e^2\epsilon_0^{-1}$, it showed the following behavior depending on the era it was calculated:

$$e^2\epsilon_0^{-1} = \begin{cases} const. & \text{if } \omega = \dfrac{1}{3} \\ t^{\frac{2}{3}} & \text{if } \omega = 0 \end{cases}$$

It is studied below with more detail for the different possibilities we have.

6.1. Case $e^2\epsilon_0^{-1}$ in the Radiation Era $\omega = \frac{1}{3}$

In this case the relationship we obtained was:

$$e^2\epsilon_0^{-1} = const.$$

from which it may be obtained that:

1. $e^2 = \epsilon_0 = const.$ Case envisaged above.
2. $e^2 = \epsilon_0$ being able to vary in any way.

3. $e^2 = \epsilon_0$ imposing the condition $\epsilon_0 = \mu_0 = \frac{1}{c}$. This condition is derived from the $TH\epsilon\mu$ formalism, devised by Lightman and Lee (see [29]) and may be used to implement the Einstein's Equivalence principle as presented by Will (see [30]).

From this relationship we obtain furthermore that:

$$e^2 = \epsilon_0 = \mu_0 = \frac{1}{c}$$

we recall that in this case the speed of the light varies as: $c \propto t^{-1/2}$.

$$e^2 = \epsilon_0 = \mu_0 \propto t^{1/2}$$

being verified furthermore

$$e^2 = \hbar \propto t^{1/2}$$

With respect to electrical and magnetic field they behave as:

$$E \propto t^{-5/4} \quad \text{and} \quad H \propto t^{-5/4}$$

$$H = E$$

the electromagnetic energy density behaved as

$$u_{EM} = \epsilon_0 E^2 + \mu_0 H^2 \propto t^{-2}$$

this result is coherent with the obtained for the radiation energy density $\rho = a\theta^4 \propto t^{-2}$. With these results the fine structure constant continuous being constant and the Bohr radius behaved as the radius of the Universe since the imposition of the condition $\epsilon_0 = \mu_0 = \frac{1}{c}$ does not alter the behavior of the constant $\hbar$.

$$R_B = \frac{\hbar^2 \epsilon_0}{e^2 m} \propto t^{1/2} \approx f(t)$$

In regard to the result $\epsilon_0 \propto t^{1/2}$ it is observed that $\epsilon_0 \propto f(t)$. This result was already explained by Møller (see [31]) and afterwards Landau and Lifshitz (see [32]) reached the same conclusion. That is also Sumners' remark (see [13]), since he sets to the result $\epsilon_0 = \mu_0 \propto t^{1/2}$ as well this coincidence, as we will see, occurs only in the case of radiation.

6.2. Case $e^2\epsilon_0^{-1}$ in the Matter Era $\omega = 0$

In this case, the relationship that we had obtained was:

$$e^2\epsilon_0^{-1} \propto t^{2/3}.$$

from this relationship it is deduced that:

1. $\epsilon_0 = const.$ and $e \propto t^{1/3}$. Case envisaged in the work.
2. $e = const.$ and $\epsilon_0^{-1} \propto t^{2/3}. \leftrightarrow \mu_0 \propto t^{4/3}$. It is remembered that in this case $c \propto t^{-1/3}$. Concerning to the quantities E and H they show the next behavior:

   $$E \propto t^{-2/3} \quad \text{and} \quad H \propto t^{-5/3}$$

   while the electromagnetic energy density behaved as:

   $$u_{EM} = \epsilon_0 E^2 + \mu_0 H^2 \propto t^{-2}$$

   in this case the constant α also continues being constant.
3. Imposing the condition $\epsilon_0 = \mu_0 = \frac{1}{c}$ ($TH\epsilon\mu$ formalism) the following results are obtained:

   $$\epsilon_0 = \mu_0 = \frac{1}{c} \,/\, e^2\epsilon_0^{-1} \propto t^{2/3}$$

   we recall that in this case $c \propto t^{-1/3}$. Then:

   $$\epsilon_0 = \mu_0 \propto t^{1/3}$$

   (the Sumner's results (see [13]) do not coincide with ours in this case) while

   $$e^2 = \hbar \propto t$$

   i.e., $e \propto t^{1/2}$. Concerning to the quantities E and H show the following behavior:

   $$E \propto t^{-7/6} \quad \text{and} \quad H \propto t^{-7/6}$$
   $$H = E$$

   while the Electromagnetic energy density behaved as

   $$u_{EM} = \epsilon_0 E^2 + \mu_0 H^2 \propto t^{-2}$$

   In this case, the constant α also continues being constant in spite of the fact that $\hbar \propto t$, i.e., our new results, do not alter at all the value of $\hbar$ already obtained and we verify the $TH\epsilon\mu$ formalism. In this case, the Bohr radius varies like the scale factor f, the radius of the Universe

   $$R_B = \frac{\hbar^2\epsilon_0}{e^2 m} \propto t^{2/3} \approx f(t)$$

   while Bohr total energy is maintained constant (result that does not surprise us since in the case of matter predominance (all) energies are preserved while the moments are not)

$$E_{TB} = \frac{me^4}{\epsilon_0^2 \hbar^2} = const.$$

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